سكشن رياضة رقم ٦

١٢/٤/١ (عيد الأم)

fx-991ES Plus

Two way power

" Elementary fn "

$\boxed{أ}$ الدالة اللوغاريتمية

$$f(z) = Ln(z) = Ln(r) + i(\theta \pm 2n\pi)$$

* Evaluate

$\boxed{1}$ $Ln(z) = Ln(\sqrt{2} + i\sqrt{6})$

sol

$$r = \sqrt{2+6} = \sqrt{8}$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{6}}{\sqrt{2}}\right) = \frac{\pi}{3}$$

$$Ln(z) = Ln(\sqrt{8}) + i\left(\frac{\pi}{3} \pm 2n\pi\right)$$

$\boxed{2}$ $3^i = e^{Ln(3^i)} = e^{i Ln(3)}$

$$= \cos(Ln(3)) + i\sin(Ln(3))$$

$\boxed{3}$ $e^{z} = 2$

$z = Ln(2) = Ln(2 + i0)$

$= Ln(2) + i(\pm 2n\pi)$ $\qquad$ $r = \sqrt{4}$

$\theta = \tan^{-1}\left(\frac{0}{2}\right) = 0$

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* show that $\frac{d}{dz}\left(Ln(z)\right) = \frac{1}{z}$

sol

$f(z) = Ln\, z = \underbrace{Ln(r)}_{i} + i\underbrace{\theta}_{v}$

$u_r = \frac{1}{r}$ $\qquad$ $-\frac{1}{r}$ $\qquad$ $u_\theta = 0$

$\qquad\qquad\qquad\qquad$ $\frac{1}{r}$

$v_r = 0$ $\qquad\qquad\qquad$ $v_\theta = 1$

$f_n \longrightarrow$ analytic

$f'(z) = (u_r + iv_r)\, e^{-i\theta} = \frac{1}{r} e^{-i\theta}$

$= \frac{1}{r e^{i\theta}}$ $\qquad$ where $re^{i\theta} = z$

$\therefore f'(z) = \frac{1}{z}$ #

*Show that $(1+i)^i = e^{-\left(\frac{\pi}{4} \pm 2n\pi\right)} \cdot e^{\frac{i}{2} Ln(2)}$

<u>L.H.S</u> $= (1+i)^i = e^{Ln(1+i)^i} = e^{i Ln(1+i)}$

$= \cos(Ln(1+i)) + i \sin[Ln(1+i)]$

$\rightarrow Ln(1+i) \qquad r = \sqrt{2}, \ \theta = \frac{\pi}{4}$

$Ln(1+i) = Ln(\sqrt{2}) + i\left(\frac{\pi}{4} \pm 2n\pi\right)$

$= e^{i\left[Ln(\sqrt{2}) + i\left(\frac{\pi}{4} \pm 2n\pi\right)\right]}$

$= e^{i[Ln(\sqrt{2})} \cdot e^{i\left(\frac{\pi}{4} \pm 2n\pi\right)]} \qquad \sqrt{2} = 2^{\frac{1}{2}}$

$= e^{i Ln(2)^{\frac{1}{2}}} \cdot e^{i\left(\frac{\pi}{4} \pm 2n\pi\right)}$

$= e^{\frac{i}{2} Ln(2)} \cdot e^{-\left(\frac{\pi}{4} \pm 2n\pi\right)}$

$\boxed{2}$ الدالة المثلثية :

$$\sin z = \frac{e^{iz} - e^{-iz}}{2i} \quad , \quad \cos z = \frac{e^{iz} + e^{-iz}}{2}$$

$$\sinh z = \frac{e^{z} - e^{-z}}{2} \quad , \quad \cosh z = \frac{e^{z} + e^{-z}}{2}$$

$$\cosh^2 - \sinh^2 = 1$$

$$\sinh(A+B) = \sinh A \cosh B + \cosh A \sinh B$$

$$\cosh(A+B) = \cosh A \cosh B + \sinh A \sinh B$$

$\boxed{1}$ show that:- $|\sin z|^2 = \sin^2 x + \sinh^2 y$

sol

$$\sin(z) = \sin(x+iy)$$

$$= \sin(x)\cos(iy) + \cos(x)\sin(iy)$$

$$= \sin(x) * \cosh y + \cos(x)\sinh(y)$$

$$|\sin z|^2 = \sin^2 x \cosh^2 y + \cos^2 x \sinh^2 y$$

$$|\sin z|^2 = \sin^2(x)\left[1+\sinh^2 y\right] + \left[1-\sin^2 x\right]\sinh^2(y)$$

$$= \sin^2 x + \sinh^2 y \quad \#$$

$\boxed{2}$ **solve** $\cosh z = \frac{1}{2}$

$$\cosh z = \frac{e^{z} + e^{-z}}{2} = \frac{1}{2} \Rightarrow e^{z} + e^{-z} = 1$$

$$e^{z} - 1 + e^{-z} = 0 \qquad * e^{z}$$

$$\underset{a}{e^{2z}} \; \underset{b}{- e^{z}} \; \underset{c}{+1} = 0$$

$$e^{z} = \frac{-b \pm \sqrt{b^2-4ac}}{2a} = \frac{1 \pm \sqrt{1-4}}{2} = \frac{1}{2} \pm i\frac{\sqrt{3}}{2}$$

$$e^{z} = \frac{1}{2} + i\frac{\sqrt{3}}{2} \qquad\qquad e^{z} = \frac{1}{2} - i\frac{\sqrt{3}}{2}$$

$$z_1 = \ln\left(\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) \qquad\qquad z_2 = \ln\left(\frac{1}{2} - i\frac{\sqrt{3}}{2}\right)$$

$$r = 1, \; \theta = \frac{\pi}{3} \qquad\qquad r = 1, \; \theta = 2\pi - \frac{\pi}{3}$$

$\cancel{z = \ln(1) + i\left(\frac{\pi}{3} \pm 2n\pi\right)}$ $z_1 =$

$$Z_1 = \ln(1) + i\left(\frac{\pi}{3} \pm 2n\pi\right)$$

$$Z_2 = \ln(1) + i\left(2\pi - \frac{\pi}{3} \pm 2n\pi\right)$$

$\boxed{3}$ show that :- $\sinh^{(-1)} z = \ln(z + \sqrt{z^2+1})$

$$\text{L.H.S} = \sinh^{(-1)} z = w$$

$$z = \sinh w = \frac{e^{w} - e^{-w}}{2}$$

$$2z = e^{w} - e^{-w}$$

$$e^{w} - 2z - e^{-w} = 0 \qquad * e^{w}$$

$$e^{2w} - 2z e^{w} - 1 = 0$$

$$e^{w} = \frac{2z \pm \sqrt{4z^2 + 4}}{2} = z \pm \sqrt{z^2+1}$$

$$w = \ln\left(z \pm \sqrt{z^2+1}\right)$$

where $w = \sinh^{(-1)} z$ //

fuzzy

* axiom of complement:-

$\boxed{1}$ $c(0) = 1$, $c(1) = 0$

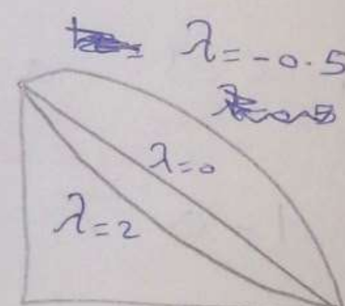
$\boxed{2}$ $a < b \longrightarrow c(a) > c(b)$

$\boxed{3}$ c is continous fn.

$\boxed{4}$ $c(c(a)) = a$

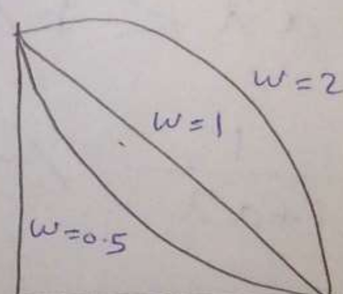
$\boxed{1}$ sugeno

$$c = \frac{1-a}{1+\lambda a} \qquad -1 < \lambda < \infty$$



$\boxed{2}$ Yager

$$c = (1-a^{w})^{\frac{1}{w}} \qquad 0 < w < \infty$$



* show that sugeno , Yager satisfied complement axioms.

solution

← الدكتور قال يكفي إنك تثبت أول شرطين.

→ **Sugeno** $c = \dfrac{1-a}{1+\lambda a}$

[1] $c(0) = 1$ ، $c(1) = 0$

[2] Let $a < b \Rightarrow -a > -b$

$1-a > 1-b \longrightarrow (1)$

$a\lambda < b\lambda \Rightarrow 1+a\lambda < 1+b\lambda$

$\dfrac{1}{1+a\lambda} > \dfrac{1}{1+b\lambda} \longrightarrow (2)$

بضرب ١ ، ٢

$\dfrac{1-a}{1+a\lambda} > \dfrac{1-b}{1+b\lambda}$

$c(a) > c(b)$ #

$\boxed{2}$ Yager $\quad c = (1 - a^{w})^{\frac{1}{w}}$

a) $c(0) = 1 \quad , \quad c(1) = (1-1)^{\frac{1}{w}} = 0$

b) $\cdot\ a < b \longrightarrow a^{w} < b^{w}$

$-a^{w} > -b^{w} \Rightarrow 1 - a^{w} > 1 - b^{w}$

$(1-a^{w})^{\frac{1}{w}} > (1-b^{w})^{\frac{1}{w}} \Rightarrow c(a) > c(b)$

$* \tilde{A} = \frac{0.1}{1} + \frac{0.2}{2} + \frac{0.6}{3}$

$\boxed{1}$ Yager Complement at $w = 2$

$\boxed{2}$ α-cut at $\alpha = 0.4 \Rightarrow w = 0.5, w = 1, w = 2$

$\boxed{\text{Sol}}$

$c = (1 - a^{2})^{\frac{1}{2}}$

هنعوض في القانون بقيم (a) ← 0.1, 0.2, 0.6

$\bar{\tilde{A}} = \frac{0.995}{1} + \frac{0.979}{2} + \frac{0.8}{3}$

at : $w = 2$

$$\tilde{A}_{0.4} = \frac{0.995}{1} + \frac{0.979}{2} + \frac{0.8}{3}$$

at $w = 0.5$ $\quad C = \left(1 - a^{0.5}\right)^2$

$$\cancel{\tilde{A} = \frac{0.468}{1} + \frac{0.71}{2} + \frac{0.5}{3}} \qquad \tilde{A}_{0.4} = \{1, 2, 3\}$$

$$\tilde{A}_{0.4} = \{1\}$$

at $w = 1$ $\quad C = (1 - a)$

$$\tilde{A} = \frac{0.9}{1} + \frac{0.8}{2} + \frac{0.4}{3}$$

$$\tilde{A}_{0.4} = \{1, 2, 3\}$$

→ fuzzy unions (s-norms)

$A \cup B$

* axioms of union

$\boxed{1}$ $s(1,1) = 1$, $s(0,a) = a$

$\boxed{2}$ $s(a,b) = s(b,a)$

$\boxed{3}$ $a < \grave{a}$, $b < \grave{b} \Rightarrow s(a,b) \leq s(\grave{a},\grave{b})$

$\boxed{4}$ $s(s(a,b),c) = s(a, s(b,c))$

$\boxed{1}$ Domb

$$s(a,b) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}} \qquad 0<\lambda<\infty$$

$\boxed{2}$ Dubois Prade

$$s(a,b) = \frac{a+b-ab-\min(a,b,1-\alpha)}{\max(1-a,1-b,\alpha)}$$

$\boxed{3}$ Yager

$$s(a,b) = \min\left[1, \left(a^{w} + b^{w}\right)^{\frac{1}{w}}\right] \qquad 0 < w < \alpha$$

* show Domb satisfied union axioms.

Sol

$s(1,1) = 1$, $s(0,a) =$

$$s(0,a) = \frac{1}{1+\left(\left(\frac{1}{a}-1\right)^{-\lambda}\right)^{-\frac{1}{\lambda}}} = \frac{1}{1+\frac{1}{a}-1} = a$$

$\longrightarrow s(a,b) = s(b,a)$

$$\frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}} = \frac{1}{1+\left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{a}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$\longrightarrow a < \grave{a}$

$$\frac{1}{a} > \frac{1}{\grave{a}} \Rightarrow \frac{1}{a}-1 > \frac{1}{\grave{a}}-1$$

$$\left(\frac{1}{a}-1\right)^{-\lambda} < \left(\frac{1}{\grave{a}}-1\right)^{-\lambda} \longrightarrow ①$$

$$b < \grave{b}$$

$$\therefore \left(\frac{1}{b}-1\right)^{-\lambda} < \left(\frac{1}{\grave{b}}-1\right)^{-\lambda} \longrightarrow ②$$

$$\left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{b}-1\right)^{-\lambda} < \left(\frac{1}{\grave{a}}-1\right)^{-\lambda} + \left(\frac{1}{\grave{b}}-1\right)^{-\lambda}$$

$$\left[\left(\frac{1}{a}-1\right)^{-\lambda} + \left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}} > \left[\left(\frac{1}{\grave{a}}-1\right)^{-\lambda} + \left(\frac{1}{\grave{b}}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\underbrace{1 +}_{\to A} \quad > \quad \underbrace{1 +}_{\to B}$$

$$\frac{1}{A} < \frac{1}{B}$$

$$\therefore s(a,b) < s(\grave{a},\grave{b}).$$

$\boxed{4}$ $s(s(a,b),c) = s(a, s(b,c))$

$$L.H.S = S(s(a,b),c) = \frac{1}{1+\left[\left(\frac{1}{s(a,b)}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$s(a,b) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{-\frac{1}{\lambda}}}$$

$$\frac{1}{s(a,b)} = 1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\frac{1}{s(a,b)}-1 =$$

$$\left(\frac{1}{s(a,b)}-1\right)^{-\lambda} = \left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}$$

$$s(s(a,b),c) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$\to$ ①

$\underline{R.H.S} = S(a, S(b,c))$

$$= \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{S(b,c)}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$S(b,c) = \frac{1}{1+\left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}$$

$$\frac{1}{S(b,c)} = 1+\left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\frac{1}{S(b,c)}-1 = \left[\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}$$

$$\left(\frac{1}{S(b,c)}-1\right)^{-\lambda} = \left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}$$

$$\boxed{S(a,S(b,c)) = \frac{1}{1+\left[\left(\frac{1}{a}-1\right)^{-\lambda}+\left(\frac{1}{b}-1\right)^{-\lambda}+\left(\frac{1}{c}-1\right)^{-\lambda}\right]^{\frac{-1}{\lambda}}}} \to (2)$$

$L.H.S = R.H.S$

$\therefore$